Honors Calculus Summer Assignment

Pinkston 2023-2024

Possible online Resources if you don't remember or know how to do a problem.

www.purplemath.com www.kutasoftware.com http://www.khanacademy.org/

You have chosen to take Honors Calculus, an accelerated course. You are expected to have a strong mathematical background and be willing to **work hard *all year* long.**

The first chapter of the textbook is referred to as Chapter Zero. Chapter Zero and the first four sections of Chapter One are basically a review of Algebra II and Pre-Calculus topics. Instead of using class time to discuss these topics, I am assigning problems to you for the summer! ☺ It is expected that you will complete the problems in this packet. A few years ago, I decided to compile problems from various websites and combine those problems with my own. **Your textbook: Calculus (an applied approach) *Larson*: Houghton Mifflin. (8th edition)**

This packet has both reference materials and problems for you to complete.

DIRECTIONS:

Complete the problems with quality. Do your best to review the material. Be prepared to ask questions the first week of school.

This packet will NOT be due on the first day of school, as it has been in years past. You will be required to submit the Summer Assignment in three submissions: dates to be determined. There will also be a Summative Summer Assignment test within the first two-three weeks of school.

It is in your best interest to complete the packet, by the start of school, with quality. It is to be considered a review of pre-requisite content.

Supplies:

- It is strongly suggested that you purchase a graphing calculator for the upcoming year. Check your local office supply store/on line. There are several on the market, however, the Texas Instruments: TI –84 and TI-84plus "family" of calculators are the preferred choice here at Brandywine. A few calculators that may NOT be used on tests and quizzes are the TI-Nspire CAS, TI – 89 and the TI-92 (Or any Casio, Hewlett-Packard, etc. equivalent).
- **Please purchase a three – ring binder, 2.5 to 3 inch**. A one - inch binder will not be large enough to last throughout the entire school year.

I may check my school email every 2-3 weeks during the summer in case you have any concerns: mary.pinkston@bsd.k12.de.us

Have a wonderful summer and I will see you in August/September! Ms. Pinkston ☺

Summer Assignment Problems:

Simplifying Radicals

An expression under a radical sign is in simplest radical form when:

1) there is no integer under the radical sign with a perfect square factor,
2) there are no fractions under the radical sign,
3) there are no radicals in the denominator

Express the following in simplest radical form.

1) $\sqrt{72}$ _____ 2) $\sqrt{242}$ _____ 3) $\sqrt{192}$ _____ 4) $\sqrt{63}$ _____ 5) $\sqrt{147}$ _____

Properties of Exponents

PROPERTY		EXAMPLE
Product of Powers	$a^m \bullet a^n = a^{m+n}$	$x^4 \bullet x^2 = x^6$
Power of a Power	$(a^m)^n = a^{m \bullet n}$	$(x^4)^2 = x^8$
Power of a Product	$(ab)^m = a^m b^m$	$(2x)^3 = 8x^3$
Negative Power	$a^{-n} = \frac{1}{a^n}$ $(a \neq 0)$	$x^{-3} = \frac{1}{x^3}$
Zero Power	$a^0 = 1$ $(a \neq 0)$	$4^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$ $(a \neq 0)$	$\frac{x^3}{x^2} = x^1 = x$
Power of Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $(b \neq 0)$	$\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$

Simplify each expression. Answers should be written using positive exponents.

1) $r^{17} \cdot r^3 \cdot g^2 \cdot g^{11}$ _____

2) $(w^6)^8$ _____

3) $m \cdot m^{-4}$ _____

4) $\frac{y^{12}}{y^8}$ _____

Simplify each expression.

5) $(3x^7)(-5x^{-4})$ _____

6) $(-9ab^2cde^0)^2$ _____

7) $\frac{-15x^7y^{-2}}{25x^{-9}y^5}$ _____

8) $\left(\frac{4x^9}{12x^4}\right)^3$ _____

Miscellaneous problems

1) State the domain, range and the zeros of: $f(x) = -4|x+9| - 5$

Domain: _____ Range: _____ Zeros: _____

2) State the domain and range of: $f(x) = \frac{x+1}{x-7}$ Domain: _____ Range: _____

3) Factor completely over the set of integers. $x^4 - 10x^2 + 9$

_____

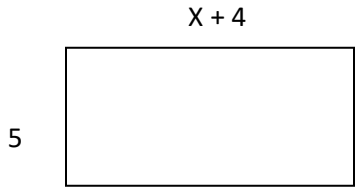
4) Write an equation of the line through (-1, -6) and (4, 8) in ***point slope*** form. _____

5) A taxicab company charges each person a flat fee of $2.65 plus an additional $.72 per quarter mile.

A. Write a linear equation find the cost for each fare. _____

B. Use the formula to find the cost for 1 person to travel 8 mi. _____

6) Find the dimensions of the rectangle given the area = 162 sq. ft.

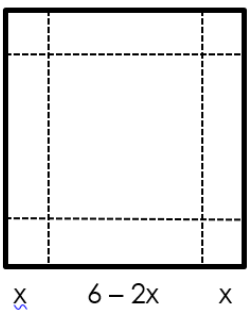


_____ x _____

7) Let $f(x) = x^2 + 2x$, $what\ is\ f(x+h)?$.

_____

8) An open box is to be made from a 6 inch square piece of material by cutting equal squares from the corners and turning up the sides.



a. Write an equation that represents the **volume.**

V(x) = _____

b. Graph the equation, V(x), using a graphing calculator (Desmos) and identify the maximum volume and at what x-value the maximum occurs.

The maximum is _____ at x = _____.

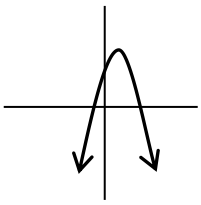
,

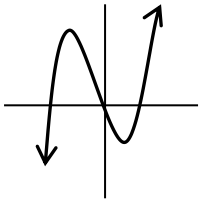
9) Let $f(x)=\sqrt{x-3}$ and $g(x)=x^2+1$. Compute: $(g\circ f)(x)=$ $g(f(x))$.

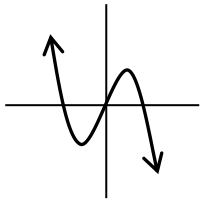
$(g\circ f)(x)$: _____

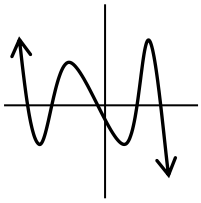
Do not simplify.

10) Which of the following could represent a complete graph of $f(x)=ax-x^3$, where a is a real number?

A. 

B. 

C. 

D. 

11) Write and equation of a **polynomial of degree three** (3) with zeros (x-intercepts): -2, 1, and 5 and going through the point $(0,-3)$. y = a(x -)(x -)(x -)

y = _____

12) The number of elk after t years in a state park is modeled by the function: $P(t)=\dfrac{1216}{1+75e^{-0.03t}}$

a) What was the initial population of elk? (Time t =0) _____

b) *When (meaning, find the "t" value) will the number of elk be 750? _____

13) Anthony invests $3500 in a savings account that pay 9% interest, compounded quarterly. If there are no other transactions, **when (t)** will his balance reach $5705?

$$A=P(1+\frac{r}{n})^{nt}.$$

_____

14) Solve the inequality: $x^2-x-12>0$. Use a number line test or graph.

A. $(-\infty,-4)\cup(3,\infty)$ B. $x=4,\ x=-3$ C. $(-3,4)$ D. $(-\infty,-3)\cup(4,\infty)$

Trigonometry (There is a Unit Circle on the last page.)

1) Fill in the table. Answers should be exact (Radical form where appropriate.) No decimals.

Degree	Radians	**SINE**	CSC	**COSINE**	SEC	**TANGENT**	COT
0							
30				$\frac{\sqrt{3}}{2}$			
45							
60							
90						Undefined	
120							
135							
150		$\frac{1}{2}$					
180							

2) Simplify: $\left(\csc(x)-\tan(x)\right)\sin(x)\cos(x)$

A. $\cos(x)-\sin^2(x)$ B. $\sin(x)-\cos^2(x)$

C. $\sin^2(x)+\cos(x)$ D. $\cos^2(x)-\sin(x)$

3) Find the **exact value** of each **without the use of a calculator – Use the Unit Circle reference sheet at the end of the packet. (No decimals. Square roots where necessary.)**

a) $\sin(3\pi)=$ _____ b) $\cos\left(-\frac{3\pi}{2}\right)=$_____ c) $\tan\left(-\frac{5\pi}{6}\right)=$ _____

d) $\csc\left(\frac{2\pi}{3}\right)=$ _____ e) $\cot\left(\frac{\pi}{2}\right)=$ _____

4) Solve the equation $2\sin^2(x)\cos(x)=\cos(x)$ algebraically. $[0,2\pi)$

X = _____

5) Find all the exact solutions to $2\sin^2(x)+3\sin(x)-2=0$ on the interval $[0,2\pi)$. (Meaning, no decimals. Write answers using square roots, where appropriate.)

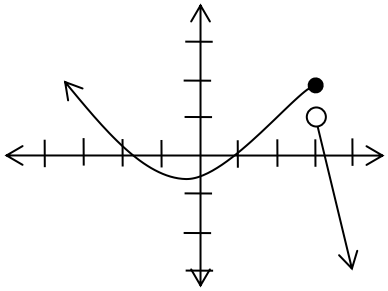
_____

Miscellaneous (2)

1) Use polynomial **long or synthetic division** to rewrite the expression $\frac{x^3+7x^2+14x-8}{x-4}$

_____

2) For the function $f(x)$ graphed answer the following



A. $f(3)$ = _____ B. $f(x)=0$ at x = _____

C. $f(0)$ = _____ D. $f(x)=1$ at x = _____

3) Give that $f(x)=\frac{2x+9}{x-8}$. Find the asymptotes (if they exist) of the function.

Vertical Asymptote(s): _____
Horizontal Asymptote(s): _____

4) Use algebra (factor) to find the exact solutions to: $15x^3-7x^2-2x=0$. Show all work.

_____

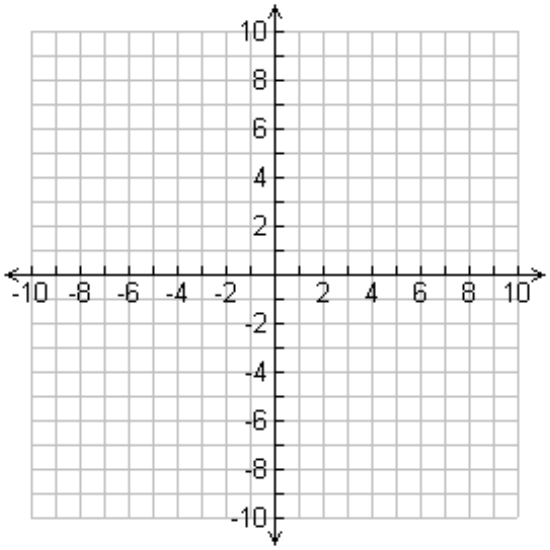
5) Solve by whichever method seems easiest. (Cross multiply?) Be sure to check for extraneous roots.

$$\frac{3}{x}=\frac{18-6x}{x^2-9}$$

_____

6) Graph the piecewise function.

$$f(x)=\begin{cases}-x^2 & -2\le x<1\\ -2 & x=1\\ 3x+5 & 1<x\le 3\end{cases}$$



7) Factor to solve the inequality. Write your answer in interval notation. $\frac{x^2-16}{x+4}\le 0$ Use a number line test or graph.

_____

8) Find the value of *k* if the line joining $(4, k)$ and $(6, 8)$ and the line joining $(-1, 4)$ and $(0, 8)$ are: (Parallel lines have the _____ slope.)

a. parallel k = _____

b. perpendicular k = _____

9) Write an equation of the **perpendicular *bisector*** (intersects at the midpoint) of the segment joining (0, 3) and (– 4, 5). (Hint: Find the slope. Find the midpoint. Draw a picture so you can see what you need to do!)

_____

10) Simplify a) $(5-i)(5+i)$ b) $(6+\sqrt{2})(6-\sqrt{2})$ c) i^2

_____ _____ _____

11) Write an equation of the quadratic function described.

a) Its graph is a parabola with x-intercepts: 2 and – 1 and y-intercept 6. $y = a(x-p)(x-q)$

_____

b) Its graph is a parabola with vertex (4, 8) and passing through the origin. $y = a(x-h)^2 + k$

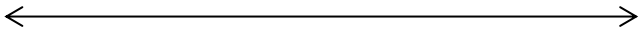
_____

12) A stone is thrown with an upward velocity of 14 m/s from a cliff 30 meters high.
The height equation, at any time "t", is given by: $h = -4.9t^2 + 14t + 30$

a) When will the stone reach its highest elevation? At t = _____ seconds

b) When will the stone hit the ground? At t = _____ seconds

13) Solve and graph. $|2x-4| \leq 5$



14) Solve for x.

a) $(8x)^{-3} = \frac{1}{64}$ x = _____ b) $8x^{-3} = \frac{1}{64}$ x = _____ c) $(\mathbf{8}+\boldsymbol{x})^{-\mathbf{3}} = \frac{\mathbf{1}}{\mathbf{64}}$ x = _____

15) Given a quadratic equation in x: $ax^2 + bx + c = 0$, State the Quadratic Formula:

_____

16) Solve the Quadratic $x^2 - 4x - 1 = 0$

a) Using the quadratic formula:

b) By completing the square:

17) **Complete the square** (once for "x" and once for "y") to write the equation in center-radius form. Give the center and radius.

$x^2 + y^2 - 2x - 8y + 16 = 0$

Center:_____

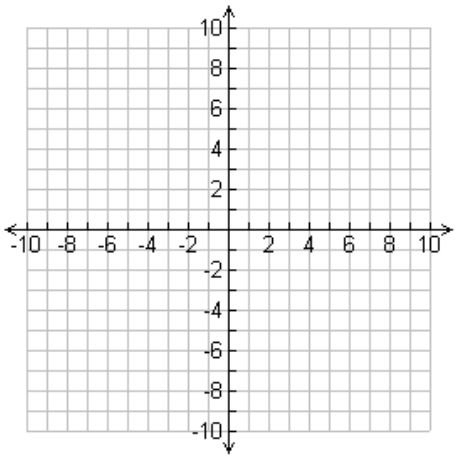
Radius:_____

18) State the domain and range of the function. Then graph the function.

$y = 2\sqrt{4 - x^2}$

Domain:_____

Range:_____



19) State the domain and the range of the function.

$y = (x - 2)^2 - 4$

Domain:_____

Range:_____

Operations With Polynomials (Algebra I)
To *add or subtract polynomials*, just combine like terms.
To multiply polynomials, multiply the numerical coefficients and apply the rules for exponents for variables.

Perform the indicated operations and simplify:

1) $(7x^2 + 4x - 3) - (-5x^2 - 3x + 2)$ (Subtract)

_____

2) $(7x - 3)(3x + 7)$ (Multiply: FOIL)

_____

3) $-2x(5x + 11)$ (Distribute)

_____

4) $(5x - 6)^2$ (FOIL)

_____

5) $(x-7)^3$ (Multiply out!)

_____

6) $2(2x+5)^2$ (FOIL first/then distribute)

_____

7) Complete each of the following:

a) Point-Slope form of a linear equation: _____

b) Slope-intercept form:_____

c) Standard form:_____

Sum and difference of cube formulas. You may need these for at least two of the following problems.

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ Remember: $(a+b)^3 \neq a^3 + b^3$

Factoring is forever!
Factor each of the following polynomial expressions (completely) over the set of integers.

1) $5x^2 - 32x - 21$ 2) $4x^2 + 20x + 9$ 3) $15x^3 - 25x^2 + 75x - 125$

4) $x^2 + 15x + 56$ 5) $28x^3 - 7x$ 6) $216x^3 + 1$

7) $12x^2 - 44x + 7$

8) $6x + 21$

9) $16x^8y^4 - 81z^4$

10) $18x^3 - 2x^2 + 27x - 3$

11) $32x^3 - 4$

12) $8x^2 + 10x - 25$

Solving systems of Equations
Solve each system of equations by either the substitution method or the linear combination (addition/ subtraction) method. Write your answer as an ordered pair. Circle your answers.

1) y = 2x + 4
-3x + y = - 9

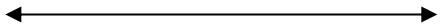
2) 2x + 3y = 6
-3x + 2y = 17

3) x – 2y = 5
3x – 5y = 8

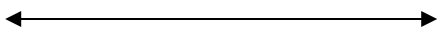
4) 3x + 7y = -1
6x + 7y = 0

Solving Linear Inequalities
Solve and graph each inequality.

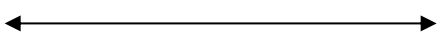
1) $-\left(\frac{3}{2}x + 18\right) \leq 6$



2) $6 < \frac{r}{2} - 7 < 20$



3) $\frac{2}{3}b - 2 > 10$ or $\frac{3}{4}b + 5 < -4$



Algebra II

1. Find $f^{-1}(x)$ if $f(x) = \sqrt[3]{4x+11}$ $f^{-1}(x)$ = _____ (inverse)

   Is $f^{-1}(x)$ a function? _____

2. a. $\log_3 x = 5$ x = _____ b. $\log_x 41 = 3$ x = _____
   Solve for x.

3. Use composition of functions (both ways: $f(g(x))$ and $g(f(x))$ circle the answer to each) to ***show that*** f and g are inverses given the following. ***Why are f & g inverses?***

   $f(x) = 6x + 1$ $g(x) = \frac{x-1}{6}$

4. State the $\mathcal{Domain}$ of each function. You may use the notation of your choice.

   a) $f(x) = 4\sqrt{x^2-9} + 12$ _____

   b) $f(x) = 2x^2 - 7$ _____ c) $f(x) = \sin(2x)$ _____ ☺

   d) $\frac{2x^2-7x+3}{(x+1)(x-2)(x+7)}$ _____

   (For + 0.5 point extra credit, also state the $\mathcal{Range}$.) _____

5. If $f(x) = x^2 - 2x$, find f(2) = _____ and find f(a + 1) = _____
   (Multiply it out!)

6. Decide whether each of the following is classified as a function of y with respect to x. Write yes or no.

a) $y^3 = 9x + 7$ _____ b) $y = \sqrt[3]{x-5}$ _____

7. Given f(x) = $\sqrt{x}$ and g(x) = $x^2 - 1$, find the composite functions indicated below, if defined:

a) f(g(1)) = _____ b) g(f(0)) = _____

c) f(g(0)) = _____ d) f(g(x)) = _____
Do not attempt to simplify.

8. A small college had 1143 students in 2004 and 2457 students in 2008. If the enrollment follows a linear growth pattern, write an equation of the line that models the situation. Then predict how many students the college will have in 2019. (Find slope. (time, #of students)

_____ _____students

9. Determine the coordinates of the x and y intercept(s) (if they exist) of the graph of each equation. If the graph does not have an x or y-intercept, write *none. Remember, that there could be more than one!!!* X-intercept (x, 0) y-intercept (0, y)

a) $y = -2x^2 + 4x - 9$ y-int _____ x-int _____

b) $y^3 = x^3 - 4x$ y-int _____ x-int _____

c) $x^2y - x^2 + 4y = -100$ y-int _____ x-int _____

10. Find x such that the **distance** between the points **is 5**. (2, -1) and (x, 2).

(Show work for credit.) $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

_____

11. Evaluate the function, h(x) at the indicated values.

$$h(x)\begin{cases} x^3 & \text{if } x < 8 \\ -\sqrt{x-7}+3 & \text{if } x > 8 \\ -x^2 & \text{if } x = 8 \end{cases}$$

a) h(π) = _____

b) h(0) = _____

c) h(-12) = _____

d) h(9) = _____

e) h(2) = _____

12. Solve for x. a. $4^x = 17$ b. $e^{x+1} = 4$

c. $Log_x 512 = 9$ d. $Log_5 x = 11$

Try this problem: Given: $f(x) = 4x - 7$, find $\frac{f(x+h)-f(x)}{h}$