PLEASE GIVE TO: ____

TO: HONORS PRE-CALCULUS STUDENTS

FROM: MRS. KIDDER

I have enclosed a page of notes and a worksheet on factoring to help you get warmed up for next year. It will be collected on the first full day of school. I have chosen this topic because factoring is the number one concept that is used in every chapter of Pre-Calculus. (As Ms. Pinkston says "Factoring is Forever!")

Please have all of the following # 1 – 149 odd (except #131) attempted for the first day of school. I (or whoever will be teaching the class) will check that you attempted them and it will count as a formative assignment. After it has been checked, you will have time in class to compare and discuss them.

Have a great summer!

Mrs Kidder

Mrs. Kidder

Math Teacher&
Department Chair

Factoring Information

Special Products

Let u and v be real numbers, variables, or algebraic expressions.

Special Product	*Example*
Sum and Difference of Same Terms	
$(u+v)(u-v) = u^2 - v^2$	$(x+4)(x-4) = x^2 - 4^2 = x^2 - 16$
Square of a Binomial	
$(u+v)^2 = u^2 + 2uv + v^2$	$(x+3)^2 = x^2 + 2(x)(3) + 3^2 = x^2 + 6x + 9$
$(u-v)^2 = u^2 - 2uv + v^2$	$(3x-2)^2 = (3x)^2 - 2(3x)(2) + 2^2 = 9x^2 - 12x + 4$
Cube of a Binomial	
$(u+v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$	$(x+2)^3 = x^3 + 3x^2(2) + 3x(2^2) + 2^3 = x^3 + 6x^2 + 12x + 8$
$(u-v)^3 = u^3 - 3u^2v + 3uv^2 - v^3$	$(x-1)^3 = x^3 - 3x^2(1) + 3x(1^2) - 1^3 = x^3 - 3x^2 + 3x - 1$

Factoring Special Polynomial Forms

Factored Form	*Example*
Difference of Two Squares	
$u^2 - v^2 = (u+v)(u-v)$	$9x^2 - 4 = (3x)^2 - 2^2 = (3x+2)(3x-2)$
Perfect Square Trinomial	
$u^2 + 2uv + v^2 = (u+v)^2$	$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2 = (x+3)^2$
$u^2 - 2uv + v^2 = (u-v)^2$	$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2 = (x-3)^2$
Sum or Difference of Two Cubes	
$u^3 + v^3 = (u+v)(u^2 - uv + v^2)$	$x^3 + 8 = x^3 + 2^3 = (x+2)(x^2 - 2x + 4)$
$u^3 - v^3 = (u-v)(u^2 + uv + v^2)$	$27x^3 - 1 = (3x)^3 - 1^3 = (3x-1)(9x^2 + 3x + 1)$

Guidelines for Factoring Polynomials

1. Factor out any common factors using the Distributive Property.
2. Factor according to one of the special polynomial forms.
3. Factor as $ax^2 + bx + c = (mx + r)(nx + s)$.
4. Factor by grouping.

Trinomials with Binomial Factors

To factor a trinomial of the form $ax^2 + bx + c$, use the following pattern.

Factors of a

$$ax^2 + bx + c = (\quad x + \quad)(\quad x + \quad)$$

Factors of c

The goal is to find a combination of factors of a and c so that the outer and inner products add up to the middle term bx. For instance, in the trinomial $6x^2 + 17x + 5$, you can write all possible factorizations and determine which one has outer and inner products that add up to $17x$.

$$(6x + 5)(x + 1),\ (6x + 1)(x + 5),\ (2x + 1)(3x + 5),\ (2x + 5)(3x + 1)$$

You can see that $(2x + 5)(3x + 1)$ is the correct factorization because the outer (O) and inner (I) products add up to $17x$.

$$(2x + 5)(3x + 1) = \overset{F}{6x^2} + \overset{O}{2x} + \overset{I}{15x} + \overset{L}{5} = 6x^2 + \overset{O+I}{17x} + 5.$$

You can draw arrows to find the correct middle term. (Encourage students to find the middle term mentally.)

$(2x - 7)(x + 15)$

$-7x$

$+30x$

$+23x$ Middle term

$(2x - 3)(x + 5)$

$-3x$

$+10x$

$+7x$ Middle term

Point out to students that "testing the middle term" means using the FOIL method to find the sum of the outer and inner terms.

Factoring by Grouping

Sometimes polynomials with more than three terms can be factored by a method called **factoring by grouping.**

Example 14 Factoring by Grouping

Use factoring by grouping to factor $x^3 - 2x^2 - 3x + 6$.

Solution

$$x^3 - 2x^2 - 3x + 6 = (x^3 - 2x^2) - (3x - 6) \qquad \text{Group terms.}$$
$$= x^2(x - 2) - 3(x - 2) \qquad \text{Factor groups.}$$
$$= (x - 2)(x^2 - 3) \qquad (x - 2) \text{ is a common factor.}$$

Checkpoint Now try Exercise 115.

FACTORING REVIEW

P.3 Exercises

Vocabulary Check

Fill in the blanks.

1. For the polynomial $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$, the degree is _______ and the leading coefficient is _______.
2. A polynomial that has all zero coefficients is called the _______.
3. A polynomial with one term is called a _______.
4. The letters in "FOIL" stand for the following.
   F_______ O_______ I_______ L_______
5. If a polynomial cannot be factored using integer coefficients, it is called _______.
6. The polynomial $u^2 + 2uv + v^2$ is called a _______.

In Exercises 1–6, match the polynomial with its description. [The polynomials are labeled (a), (b), (c), (d), (e), and (f).]

(a) $6x$ (b) $1 - 4x^3$

(c) $x^3 + 2x^2 - 4x + 1$ (d) 7

(e) $-3x^5 + 2x^3 + x$ (f) $\frac{3}{4}x^4 + x^2 + 14$

1. A polynomial of degree zero
2. A trinomial of degree five
3. A binomial with leading coefficient -4
4. A monomial of positive degree
5. A trinomial with leading coefficient $\frac{3}{4}$
6. A third-degree polynomial with leading coefficient 1

In Exercises 7–10, write a polynomial that fits the description. (There are many correct answers.)

7. A third-degree polynomial with leading coefficient -2
8. A fifth-degree polynomial with leading coefficient 8
9. A fourth-degree polynomial with a negative leading coefficient
10. A third-degree trinomial with an even leading coefficient

In Exercises 11–16, write the polynomial in standard form. Then identify the degree and leading coefficient of the polynomial.

11. $3x + 4x^2 + 2$
12. $x^2 - 4 - 3x^4$
13. $1 + x^7$
14. $-21x$
15. $1 - x + 6x^4 - 2x^5$
16. $7 + 8x$

In Exercises 17–20, determine whether the expression is a polynomial. If so, write the polynomial in standard form.

17. $7x - 2x^3 + 10$
18. $4x^3 + x - x^{-1}$
19. $\sqrt{x^2 - x^4}$
20. $\dfrac{x^2 + 2x - 3}{6}$

In Exercises 21–36, perform the operations and write the result in standard form.

21. $(6x + 5) - (8x + 15)$
22. $(2x^2 + 1) - (x^2 - 2x + 1)$
23. $-(x^3 - 2) + (4x^3 - 2x)$
24. $-(5x^2 - 1) - (-3x^2 + 5)$
25. $(15x^2 - 6) - (-8.1x^3 - 14.7x^2 - 17)$
26. $(15.6x^4 - 18x - 19.4) - (13.9x^4 - 9.2x + 15)$
27. $3x(x^2 - 2x + 1)$
28. $y^2(4y^2 + 2y - 3)$
29. $-5z(3z - 1)$
30. $(-3x)(5x + 2)$
31. $(1 - x^3)(4x)$
32. $-4x(3 - x^3)$
33. $(2.5x^2 + 5)(-3x)$
34. $(2 - 3.5y)(4y^3)$
35. $-2x\left(\frac{1}{8}x + 3\right)$
36. $6y\left(4 - \frac{3}{8}y\right)$

In Exercises 37–68, multiply or find the special product.

37. $(x + 3)(x + 4)$
38. $(x - 5)(x + 10)$
39. $(3x - 5)(2x + 1)$
40. $(7x - 2)(4x - 3)$
41. $(2x - 5y)^2$
42. $(5 - 8x)^2$
43. $(x + 10)(x - 10)$
44. $(2x + 3)(2x - 3)$
45. $(x + 2y)(x - 2y)$
46. $(2x + 3y)(2x - 3y)$

47. $(2r^2 - 5)(2r^2 + 5)$
48. $(3a^3 - 4b^2)(3a^3 + 4b^2)$
49. $(x + 1)^3$
50. $(x - 2)^3$
51. $(2x - y)^3$
52. $(3x + 2y)^3$
53. $(\frac{1}{2}x - 5)^2$
54. $(\frac{3}{5}t + 4)^2$
55. $(\frac{1}{4}x - 3)(\frac{1}{4}x + 3)$
56. $(2x + \frac{1}{6})(2x - \frac{1}{6})$
57. $(2.4x + 3)^2$
58. $(1.8y - 5)^2$
59. $(-x^2 + x - 5)(3x^2 + 4x + 1)$
60. $(x^2 + 3x + 2)(2x^2 - x + 4)$
61. $[(m - 3) + n][(m - 3) - n]$
62. $[(x + y) + 1][(x + y) - 1]$
63. $[(x - 3) + y]^2$
64. $[(x + 1) - y]^2$
65. $5x(x + 1) - 3x(x + 1)$
66. $(2x - 1)(x + 3) + 3(x + 3)$
67. $(u + 2)(u - 2)(u^2 + 4)$
68. $(x + y)(x - y)(x^2 + y^2)$

In Exercises 69–74, factor out the common factor.

69. $2x + 8$
70. $5y - 30$
71. $2x^3 - 6x$
72. $4x^3 - 6x^2 + 12x$
73. $3x(x - 5) + 8(x - 5)$
74. $(5x - 4)^2 + (5x - 4)$

In Exercises 75–82, factor the difference of two squares.

75. $x^2 - 64$
76. $x^2 - 81$
77. $32y^2 - 18$
78. $4 - 36y^2$
79. $4x^2 - \frac{1}{9}$
80. $\frac{25}{36}y^2 - 49$
81. $(x - 1)^2 - 4$
82. $25 - (z + 5)^2$

In Exercises 83–90, factor the perfect square trinomial.

83. $x^2 - 4x + 4$
84. $x^2 + 10x + 25$
85. $x^2 + x + \frac{1}{4}$
86. $x^2 - \frac{4}{3}x + \frac{4}{9}$
87. $4t^2 + 4t + 1$
88. $9x^2 - 12x + 4$
89. $9t^2 + \frac{3}{2}t + \frac{1}{16}$
90. $4t^2 + \frac{8}{5}t + \frac{4}{25}$

In Exercises 91–100, factor the sum or difference of cubes.

91. $x^3 - 8$
92. $x^3 + 27$
93. $y^3 + 216$
94. $z^3 - 125$
95. $x^3 - \frac{8}{27}$
96. $x^3 + \frac{8}{125}$
97. $8x^3 - 1$
98. $27x^3 + 8$
99. $\frac{1}{8}x^3 + 1$
100. $\frac{27}{64}x^3 - 1$

In Exercises 101–114, factor the trinomial.

101. $x^2 + x - 2$
102. $x^2 + 5x + 6$
103. $s^2 - 5s + 6$
104. $t^2 - t - 6$
105. $20 - y - y^2$
106. $24 + 5z - z^2$
107. $3x^2 - 5x + 2$
108. $3x^2 + 13x - 10$
109. $2x^2 - x - 1$
110. $2x^2 - x - 21$
111. $5x^2 + 26x + 5$
112. $8x^2 - 45x - 18$
113. $-5u^2 - 13u + 6$
114. $-6x^2 + 23x + 4$

In Exercises 115–118, factor by grouping.

115. $x^3 - x^2 + 2x - 2$
116. $x^3 + 5x^2 - 5x - 25$
117. $6x^2 + x - 2$
118. $3x^2 + 10x + 8$

In Exercises 119–150, completely factor the expression.

119. $x^3 - 16x$
120. $12x^2 - 48$
121. $x^3 - x^2$
122. $6x^2 - 54$
123. $x^2 - 2x + 1$
124. $9x^2 - 6x + 1$
125. $1 - 4x + 4x^2$
126. $16 - 6x - x^2$
127. $2x^2 + 4x - 2x^3$
128. $7y^2 + 15y - 2y^3$
129. $9x^2 + 10x + 1$
130. $13x + 6 + 5x^2$
131. $\frac{1}{8}x^2 - \frac{1}{96}x - \frac{1}{16}$
132. $\frac{1}{81}x^2 + \frac{2}{9}x - 8$
133. $3x^3 + x^2 + 15x + 5$
134. $5 - x + 5x^2 - x^3$
135. $3u - 2u^2 + 6 - u^3$
136. $x^4 - 4x^3 + x^2 - 4x$
137. $25 - (z + 5)^2$
138. $(t - 1)^2 - 49$
139. $(x^2 + 1)^2 - 4x^2$
140. $(x^2 + 8)^2 - 36x^2$
141. $2t^3 - 16$
142. $5x^3 + 40$
143. $4x(2x - 1) + 2(2x - 1)^2$
144. $5(3 - 4x)^2 - 8(3 - 4x)(5x - 1)$
145. $2(x + 1)(x - 3)^2 - 3(x + 1)^2(x - 3)$
146. $7(3x + 2)^2(1 - x)^2 + (3x + 2)(1 - x)^3$
147. $7x(2)(x^2 + 1)(2x) - (x^2 + 1)^2(7)$
148. $3(x - 2)^2(x + 1)^4 + (x - 2)^3(4)(x + 1)^3$
149. $2x(x - 5)^4 - x^2(4)(x - 5)^3$

Factoring

Always look for a Greatest Common Factor FIRST!!!

2 TERMS

(Must be in one of the following forms to factor with two terms)

Difference of Two Perfect Squares

$a^2 - b^2 = (a + b)(a - b)$

OR

Sum Of Cubes

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

OR

Difference of Cubes

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

3 TERMS

<u>Example</u>: $2x^2 + 3x - 5$

1. Find two numbers that multiply to -10 (2 * -5) and add to 3.
   $\underline{5} \times \underline{-2} = -10$ and $\underline{5} + \underline{-2} = 3$
   The numbers are 5 & -2
2. **<u>By Grouping</u>**: change the middle term "+3x" to " - 2x + 5x" and then factor by grouping
   $2x^2 - 2x + 5x - 5$ or $2x(x - 1) + 5(x - 1)$ or
   $(2x + 5)(x - 1)$
3. **<u>Alternative Method</u>**: write the leading coefficient of 2x in the front of both terms
   Then put in the two numbers you found
   $(2x \underline{+5})(2x \underline{-2})$
   Divide out what the terms have in common
   $(2x + 5)$ nothing in common so keep the same
   $(2x - 2)$ divide by 2 = $(x - 1)$
   Answer: $(2x + 5)(x - 1)$

4 TERMS

(Grouping)

Group first two and last two terms and see if each pair has a G.C.F. (May need to change order of the terms)

$\underline{2x^3 - 8x^2} + \underline{3x - 12}$

THEN

If the G.C.F. of each pair results in a common binomial, factor out the binomial..

$2x^2(x - 4) + 3(x - 4)$

THEN

Write the binomial times the binomial created by the terms left when GCF. binomial was pulled out.

$(x - 4)(2x^2 + 3)$

1. If nothing can be done to the original expression, then it is PRIME 2. Check to see if any of your final answers will factor further. 3. Check your answer by multiplying.