

Honors Calculus Summer Assignment

Pinkston 2022-2023



Possible online Resources if you don't remember or know how to do a problem.

www.purplemath.com

www.kutasoftware.com

<http://www.khanacademy.org/>

You have chosen to take Honors Calculus, an accelerated course. You are expected to have a strong mathematical background and be willing to **work hard all year long**.

The first chapter of the textbook is referred to as Chapter Zero. Chapter Zero and the first four sections of Chapter One are basically a review of Algebra II and Pre-Calculus topics. Instead of using class time to discuss these topics, I am assigning problems to you for the summer! ☺ It is expected that you will complete the problems in this packet. A few years ago, I decided to compile problems from various websites and combine those problems with my own. **Your textbook: Calculus (an applied approach) Larson: Houghton Mifflin. (8th edition)**

This packet has both reference materials and problems for you to complete.

DIRECTIONS:

Complete the problems with quality. Do your best to review the material. Be prepared to ask questions the first week of school.

This packet will **NOT** be due on the first day of school, as it has been in years past. **You will be required to submit the Summer Assignment in three submissions: dates to be determined.** There will also be a Summative Summer Assignment test within the first two-three weeks of school.

It is in your best interest to complete the packet, by the start of school, with quality. It is to be considered a review of pre-requisite content.

Supplies:

- It is strongly suggested that you purchase a graphing calculator for the upcoming year. Check your local office supply store/on line. There are several on the market, however, the Texas Instruments: TI-84 and TI-84plus "family" of calculators are the preferred choice here at Brandywine. A few calculators that may NOT be used on tests and quizzes are the TI-Nspire CAS, TI-89 and the TI-92 (Or any Casio, Hewlett-Packard, etc. equivalent).
- **Please purchase a three – ring binder, 2.5 to 3 inch.** A one - inch binder will not be large enough to last throughout the entire school year.

I may check my school email every 2-3 weeks during the summer in case you have any concerns:
mary.pinkston@bsd.k12.de.us

Have a wonderful summer and I will see you in August/September! Ms. Pinkston ☺

Summer Assignment Problems:

Simplifying Radicals

An expression under a radical sign is in simplest radical form when:

- 1) there is no integer under the radical sign with a perfect square factor,
- 2) there are no fractions under the radical sign,
- 3) there are no radicals in the denominator

Express the following in simplest radical form.

1) $\sqrt{72}$ _____ 2) $\sqrt{242}$ _____ 3) $\sqrt{192}$ _____ 4) $\sqrt{63}$ _____ 5) $\sqrt{147}$ _____

Properties of Exponents

PROPERTY		EXAMPLE
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$x^4 \cdot x^2 = x^6$
Power of a Power	$(a^m)^n = a^{m \cdot n}$	$(x^4)^2 = x^8$
Power of a Product	$(ab)^m = a^m b^m$	$(2x)^3 = 8x^3$
Negative Power	$a^{-n} = \frac{1}{a^n} \quad (a \neq 0)$	$x^{-3} = \frac{1}{x^3}$
Zero Power	$a^0 = 1 \quad (a \neq 0)$	$4^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$	$\frac{x^3}{x^2} = x^1 = x$
Power of Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (b \neq 0)$	$\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$

Simplify each expression. Answers should be written using positive exponents.

1) $r^{17} \cdot r^3 \cdot g^2 \cdot g^{11}$ _____ 2) $(w^6)^8$ _____

3) $m \cdot m^{-4}$ _____ 4) $\frac{y^{12}}{y^8}$ _____

Simplify each expression.

5) $(3x^7)(-5x^{-4})$ _____ 6) $(-9ab^2cde^0)^2$ _____

7) $\frac{-15x^7y^{-2}}{25x^{-9}y^5}$ _____ 8) $\left(\frac{4x^9}{12x^4}\right)^3$ _____

Miscellaneous problems

1) State the domain, range and the zeros of: $f(x) = -4|x + 9| - 5$

Domain: _____ Range: _____ Zeros: _____

2) State the domain and range of: $f(x) = \frac{x+1}{x-7}$ Domain: _____ Range: _____

3) Factor completely over the set of integers. $x^4 - 10x^2 + 9$

4) Write an equation of the line through $(-1, -6)$ and $(4, 8)$ in **point slope** form. _____

5) A taxicab company charges each person a flat fee of \$2.65 plus an additional \$.72 per quarter mile.

A. Write a linear equation find the cost for each fare. _____

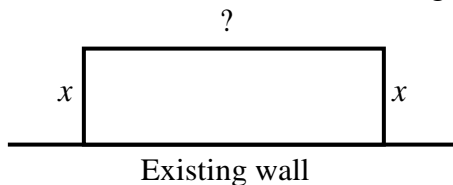
B. Use the formula to find the cost for 1 person to travel 8 mi. _____

6) Find the dimensions of the rectangle given the area = 162 sq. ft.



7) Let $f(x) = x^2 + 2x$, what is $f(x + h)$? .

8) Three sides of a fence and an existing wall form a rectangular enclosure. The total length of a fence used for the three sides is 240 ft. Let x be the length of the two sides perpendicular to the wall as shown. **Write an equation of area A of the enclosure as a function of the length x** of the rectangular area as shown in the above figure. The find value(s) of x for which the area is 5500 ft².



$x =$ _____

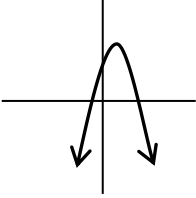
- 9) Let $f(x) = \sqrt{x-3}$ and $g(x) = x^2 + 1$. Compute: $(g \circ f)(x) = g(f(x))$.

$$(g \circ f)(x) : \underline{\hspace{10em}}$$

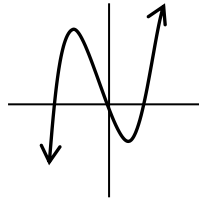
Do not simplify.

- 10) Which of the following could represent a complete graph of $f(x) = ax - x^3$, where a is a real number?

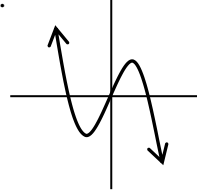
A.



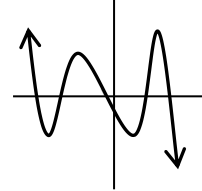
B.



C.



D.



- 11) Write an equation of a **polynomial of degree three** (3) with zeros (x-intercepts): -2, 1, and 5 and going through the point $(0, -3)$. $y = a(x - \quad)(x - \quad)(x - \quad)$

$$y = \underline{\hspace{10em}}$$

- 12) The number of elk after t years in a state park is modeled by the function: $P(t) = \frac{1216}{1 + 75e^{-0.03t}}$

- a) What was the initial population of elk? (Time $t = 0$) $\underline{\hspace{10em}}$
- b) *When (meaning, find the "t" value) will the number of elk be 750? $\underline{\hspace{10em}}$

- 13) Anthony invests \$3500 in a savings account that pay 9% interest, compounded quarterly. If there are no other transactions, **when (t)** will his balance reach \$5705?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$\underline{\hspace{10em}}$

- 14) Solve the inequality: $x^2 - x - 12 > 0$. Use a number line test or graph.

- A. $(-\infty, -4) \cup (3, \infty)$ B. $x = 4, x = -3$ C. $(-3, 4)$ D. $(-\infty, -3) \cup (4, \infty)$

Trigonometry

- 1) Fill in the table. Answers should be exact (Radical form where appropriate.) No decimals.

Degree	Radians	SINE	CSC	COSINE	SEC	TANGENT	COT
0							
30				$\frac{\sqrt{3}}{2}$			
45							
60							
90						Undefined	
120							
135							
150		$\frac{1}{2}$					
180							

- 2) Simplify: $(\csc(x) - \tan(x))\sin(x)\cos(x)$

A. $\cos(x) - \sin^2(x)$

B. $\sin(x) - \cos^2(x)$

C. $\sin^2(x) + \cos(x)$

D. $\cos^2(x) - \sin(x)$

- 3) Find the **exact value** of each **without the use of a calculator – Use the Unit Circle reference sheet at the end of the packet. (No decimals. Square roots where necessary.)**

a) $\sin(3\pi) = \underline{\hspace{2cm}}$ b) $\cos\left(-\frac{3\pi}{2}\right) = \underline{\hspace{2cm}}$ c) $\tan\left(-\frac{5\pi}{6}\right) = \underline{\hspace{2cm}}$

d) $\csc\left(\frac{2\pi}{3}\right) = \underline{\hspace{2cm}}$ e) $\cot\left(\frac{\pi}{2}\right) = \underline{\hspace{2cm}}$

- 4) Solve the equation $2\sin^2(x)\cos(x) = \cos(x)$ algebraically. $[0, 2\pi)$

$X = \underline{\hspace{4cm}}$

- 5) Find all the exact solutions to $2\sin^2(x) + 3\sin(x) - 2 = 0$ on the interval $[0, 2\pi)$. (Meaning, no decimals. Write answers using square roots, where appropriate.)

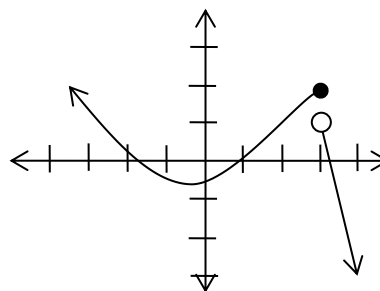
$\underline{\hspace{4cm}}$

Miscellaneous (2)

- 1) Use polynomial **long or synthetic division** to rewrite the expression $\frac{x^3 + 7x^2 + 14x - 8}{x - 4}$

- 2) For the function $f(x)$ graphed answer the following

- A. $f(3)$ B. $f(x) = 0$
 C. $f(0)$ D. $f(x) = 1$



- 3) Give that $f(x) = \frac{2x+9}{x-8}$. Find the asymptotes (if they exist) of the function.

Vertical Asymptote(s): _____

Horizontal Asymptote(s): _____

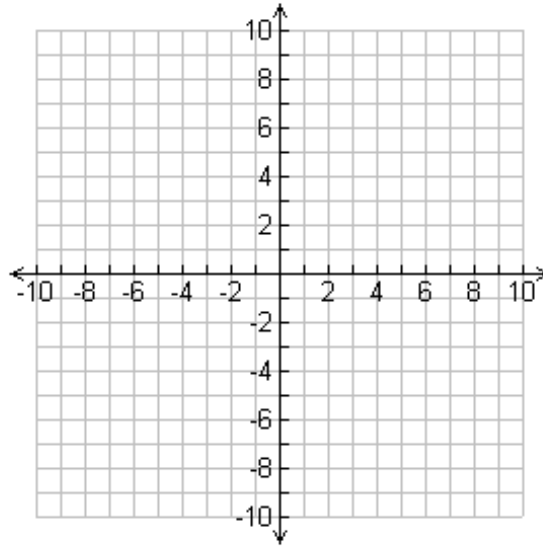
- 4) Use algebra (factor) to find the exact solutions to: $15x^3 - 7x^2 - 2x = 0$. Show all work.

- 5) Solve by whichever method seems easiest. (Cross multiply?) Be sure to check for extraneous roots.

$$\frac{3}{x} = \frac{18-6x}{x^2-9}$$

6) Graph the piecewise function.

$$f(x) = \begin{cases} -x^2 & -2 \leq x < 1 \\ -2 & x = 1 \\ 3x + 5 & 1 < x \leq 3 \end{cases}$$



7) Factor to solve the inequality. Write your answer in interval notation. $\frac{x^2 - 16}{x + 4} \leq 0$ Use a number line test or graph.

8) Find the value of k if the line joining $(4, k)$ and $(6, 8)$ and the line joining $(-1, 4)$ and $(0, 8)$ are: (Parallel lines have the _____ slope.)

a. parallel $k =$ _____

b. perpendicular $k =$ _____

9) Write an equation of the **perpendicular bisector** (intersects at the midpoint) of the segment joining $(0, 3)$ and $(-4, 5)$. (Hint: Find the slope. Find the midpoint. Draw a picture so you can see what you need to do!)

10) Simplify a) $(5 - i)(5 + i)$ b) $(6 + \sqrt{2})(6 - \sqrt{2})$ c) i^2

11) Write an equation of the quadratic function described.

a) Its graph is a parabola with x-intercepts: 2 and -1 and y-intercept 6. $y = a(x - p)(x - q)$

b) Its graph is a parabola with vertex (4, 8) and passing through the origin. $y = a(x - h)^2 + k$

12) A stone is thrown with an upward velocity of 14 m/s from a cliff 30 meters high.

The height equation, at any time "t", is given by: $h = -4.9t^2 + 14t + 30$

a) When will the stone reach its highest elevation? At t = _____ seconds

b) When will the stone hit the ground? At t = _____ seconds

13) Solve and graph. $|2x - 4| \leq 5$

←—————→

14) Solve for x.

a) $(8x)^{-3} = \frac{1}{64}$ x = _____ b) $8x^{-3} = \frac{1}{64}$ x = _____ c) $(8 + x)^{-3} = \frac{1}{64}$ x = _____

15) Given a quadratic equation in x: $ax^2 + bx + c = 0$, State the Quadratic Formula:

16) Solve the Quadratic $x^2 - 4x - 1 = 0$

a) Using the quadratic formula:

b) By completing the square:

17) **Complete the square** (once for "x" and once for "y") to write the equation in center-radius form. Give the center and radius.

$$x^2 + y^2 - 2x - 8y + 16 = 0$$

Center: _____

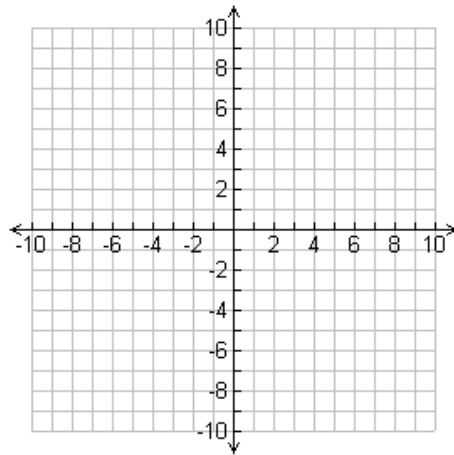
Radius: _____

18) State the domain and range of the function. Then graph the function.

$$y = 2\sqrt{4 - x^2}$$

Domain: _____

Range: _____



19) State the domain and the range of the function.

$$y = (x - 2)^2 - 4$$

Domain: _____

Range: _____

Operations With Polynomials (Algebra I)

To *add or subtract polynomials*, just combine like terms.

To *multiply polynomials*, multiply the numerical coefficients and apply the rules for exponents for variables.

Perform the indicated operations and simplify:

1) $(7x^2 + 4x - 3) - (-5x^2 - 3x + 2)$ (Subtract)

2) $(7x - 3)(3x + 7)$ (Multiply: FOIL)

3) $-2x(5x + 11)$ (Distribute)

4) $(5x - 6)^2$ (FOIL)

5) $(x - 7)^3$ (Multiply out!)

6) $2(2x + 5)^2$ (FOIL first/then distribute)

7) Complete each of the following:

a) Point-Slope form of a linear equation: _____

b) Slope-intercept form: _____

c) Standard form: _____

Sum and difference of cube formulas. You may need these for at least two of the following problems.

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2) \quad \text{Remember: } (a + b)^3 \neq a^3 + b^3$$

Factoring is forever!

Factor each of the following polynomial expressions (completely) over the set of integers.

1) $5x^2 - 32x - 21$

2) $4x^2 + 20x + 9$

3) $15x^3 - 25x^2 + 75x - 125$

4) $x^2 + 15x + 56$

5) $28x^3 - 7x$

6) $216x^3 + 1$

7) $12x^2 - 44x + 7$

8) $6x + 21$

9) $16x^8y^4 - 81z^4$

10) $18x^3 - 2x^2 + 27x - 3$

11) $32x^3 - 4$

12) $8x^2 + 10x - 25$

Solving systems of Equations

Solve each system of equations by either the substitution method or the linear combination (addition/subtraction) method. Write your answer as an ordered pair. Circle your answers.

1)
$$\begin{aligned} y &= 2x + 4 \\ -3x + y &= -9 \end{aligned}$$

2)
$$\begin{aligned} 2x + 3y &= 6 \\ -3x + 2y &= 17 \end{aligned}$$

3)
$$\begin{aligned} x - 2y &= 5 \\ 3x - 5y &= 8 \end{aligned}$$

4)
$$\begin{aligned} 3x + 7y &= -1 \\ 6x + 7y &= 0 \end{aligned}$$

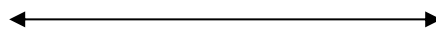
Solving Linear Inequalities

Solve and graph each inequality.

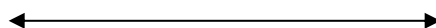
1) $-\left(\frac{3}{2}x + 18\right) \leq 6$



2) $6 < \frac{r}{2} - 7 < 20$



3) $\frac{2}{3}b - 2 > 10$ or $\frac{3}{4}b + 5 < -4$



Algebra II

1. Find $f^{-1}(x)$ if $f(x) = \sqrt[3]{4x+11}$ $f^{-1}(x) =$ _____ (inverse)

Is $f^{-1}(x)$ a function? _____

2. $\log_3 x = 5$ $x =$ _____

3. Use composition of functions (both ways: $f(g(x))$ and $g(f(x))$) circle the answer to each) to **show that** f and g are inverses given the following. **Why are f & g inverses?**

$$f(x) = 6x + 1 \qquad g(x) = \frac{x-1}{6}$$

4. State the *Domain* of each function. You may use the notation of your choice.

a) $f(x) = 4\sqrt{x^2-9} + 12$ _____

b) $f(x) = 2x^2 - 7$ _____ c) $f(x) = \sin(2x)$ _____ ☺

d) $\frac{2x^2 - 7x + 3}{(x+1)(x-2)(x+7)}$ _____

(For + 0.5 point extra credit, also state the *Range*.) _____

5. If $f(x) = x^2 - 2x$, find $f(2) =$ _____ and find $f(a+1) =$ _____
(Multiply it out!)

6. Decide whether each of the following is classified as a function of y with respect to x . Write yes or no.

a) $y^3 = 9x + 7$ _____ b) $y = \sqrt[3]{x - 5}$ _____

7. Given $f(x) = \sqrt{x}$ and $g(x) = x^2 - 1$, find the composite functions indicated below, if defined:

a) $f(g(1)) =$ _____ b) $g(f(0)) =$ _____

c) $f(g(0)) =$ _____ d) $f(g(x)) =$ _____
Do not attempt to simplify.

8. A small college had 1143 students in 2004 and 2457 students in 2008. If the enrollment follows a linear growth pattern, write an equation of the line that models the situation. Then predict how many students the college will have in 2019. (Find slope. (time, #of students))

_____ students

9. Determine the coordinates of the x and y intercept(s) (if they exist) of the graph of each equation. If the graph does not have an x or y -intercept, write *none*. Remember, that there could be more than one!!! X -intercept $(x, 0)$ y -intercept $(0, y)$

a) $y = -2x^2 + 4x - 9$ y -int _____ x -int _____

b) $y^3 = x^3 - 4x$ y -int _____ x -int _____

c) $x^2y - x^2 + 4y = -100$ y -int _____ x -int _____

10. Find x such that the **distance** between the points **is 5**. $(2, -1)$ and $(x, 2)$.

(Show work for credit.) $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

11. Evaluate the function, $h(x)$ at the indicated values.

$$h(x) \begin{cases} x^3 & \text{if } x < 8 \\ -\sqrt{x-7} + 3 & \text{if } x > 8 \\ -x^2 & \text{if } x = 8 \end{cases}$$

a) $h(\pi) = \underline{\hspace{2cm}}$

b) $h(0) = \underline{\hspace{2cm}}$

c) $h(-12) = \underline{\hspace{2cm}}$

d) $h(9) = \underline{\hspace{2cm}}$

e) $h(2) = \underline{\hspace{2cm}}$

Try this problem: Given: $f(x) = 4x - 7$, find $\frac{f(x+h)-f(x)}{h}$

